MODELING HEAT TRANSFER I A GLOW DISCHARGE IN THE STATIONARY MODE

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The temperature field of a stationary glow discharge in a gas-discharge chamber is investigated by using modelling of the heat transfer processes. The solution of the problem was obtained on the basis of an asymptotic expansion of the equations.

Definite successes have been achieved in recent years in the area of producing electrical discharge lasers [1], however, up to now the theoretically computed powers have not been successfully yielded. The main obstacle is the development of different kinds of instabilities in the glow discharge and, consequently, the transition of the glow into an arc discharge. The temperature-overheating instability [2, 3] is most widespread. The present paper is devoted to clarification of one of the possible reasons for its origin.

COMPUTATION SCHEME

Let us consider the heat transfer in a discharge gap under the assumption of stationarity of the gasdynamic flow and heat transfer. The rational basis for the general construction of the gasdynamic contour is the transverse pumping scheme for which the discharge current and the stream velocity are oriented perpendicularly to the laser optical axis [4]. The gas-discharge chamber (GDC) section transverse to the flow is a strongly elongated rectangle that permits considering the problem quasiplanar. Let us make still another simplifying assumption. We shall consider the velocity vector of the gas medium to have just one component u directed along the OX axis. We direct the OY axis transverse to the flow. We consider the working mixture as an ideal gas. In this case the equations describing the process take the following form:

Motion

$$\rho u \frac{\partial u}{\partial x} = -\frac{\partial P}{\partial x} + \frac{4}{3}\frac{\partial}{\partial x}\mu\frac{\partial u}{\partial x} + \frac{\partial}{\partial y}\mu\frac{\partial u}{\partial y},$$

$$0 = -\frac{\partial P}{\partial y} + \frac{1}{3}\frac{\partial}{\partial x}\mu\frac{\partial u}{\partial y}, \qquad (1)$$

(the Stokes hypothesis about the relation between the shear and volume viscosity is used here),

Energy

$$\rho c_P u \frac{\partial T}{\partial x} = \frac{\partial}{\partial x}\lambda\frac{\partial T}{\partial x} + \frac{\partial}{\partial y}\lambda\frac{\partial T}{\partial y} + \mu\Phi + q, \qquad (2)$$

where

$$\Phi = \frac{4}{3}\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2,$$

Continuity

$$\frac{\partial(\rho u)}{\partial x} = 0, \qquad (3)$$

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· Perfect gas state

$$P = R^{*}\rho T. \tag{4}$$

Let us examine the processes described by these equalities in greater detail. The typical pumping rates for flow-through lasers are 20-50 m/sec. The discharge chamber size along the optical axis is $\gtrsim 1$ m, the width 0.1 m, the discharge zone dimension along the stream is 0.1-0.5 m, and the gas mixture pressure is 2660-6650 Pa. The gas mixture temperature at the entrance to the GDC is usually room temperature. Helium in large quantity is present in the working mixture. In this case the Mach number at the GDC entrance will be in the $0.02 \leq M_0 \leq 0.1$ range, i.e., the gas mixture can be considered incompressible for isothermal flow [5]. Let us introduce the dimensionless pressure $\overline{P} = (P - P_0)/(\rho_0 u_0^2)$, then (4) can be represented in the form

$$\rho T = \rho_0 T_0 (1 + M_0^2 \gamma_0 \overline{P}), \tag{5}$$

where $\gamma_0 = c_P/c_V$ and all the quantities with subscript 0 are taken at the GDC entrance. We have therefore obtained a formal expansion of the equation of state in the Mach number. It can be considered to M_0^2 accuracy that

$$\rho T = A = \text{const}. \tag{6}$$

For further simplification of the problem we assume that μ, λ, c_P are independent of the temperature in the temperature band. Now, using the boundary layer approximation and the condition $M_0 << 1$ we obtain [6]

$$\rho u \frac{\partial u}{\partial x} = -\frac{\partial P}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2}, \quad \frac{\partial P}{\partial y} = \frac{\mu}{3} \frac{\partial^2 u}{\partial x \partial y},$$
$$\rho c_P u \frac{\partial T}{\partial x} = \lambda \frac{\partial^2 T}{\partial y^2} + q. \tag{7}$$

The system (3), (6) and (7) is closed. Substituting the appropriate boundary conditions, we can solve it and obtain the velocity and temperature field distributions. It is sufficiently complicated to solve such a system, consequently, we try to obtain an approximate solution.

We consider, for the approximate characteristics of the solutions, that the energy delivered by using a heat source is much less than the initial energy per unit mass of the gas mixture. Indeed, the initial temperatue is $T_0 \approx 300$ K while heating is realized at $\Delta T_0 \lesssim 100$ K therefore, $\Delta T/T_0 \lesssim 0.3$. Let us represent the source function in the form of the asymptotic series

$$q = q_0 + q_1 + q_2 + \ldots, \tag{8}$$

where $q_0 = 0$ and the remaining terms satisfy the inequalities $q_1 >> q_2 >> q_3 \ldots$ Moreover, the inequality $q_1 d/(\rho_0 c_P u_0 T_0) << 1$ is satisfied for q_1. We represent all the remaining functions analogously to (8). Now retaining only the zeroth terms, we obtain the following equations for the isothermal flow

$$\frac{\partial u_0}{\partial x} = 0, \quad \rho_0 = \text{const}, \quad T_0 = \text{const}, \quad \frac{\partial P_0}{\partial y} = 0, \quad \frac{\partial P_0}{\partial x} = \mu \frac{\partial^2 u_0}{\partial y^2}. \tag{9}$$

The solution has the form

$$\frac{dP_0}{dx} = -B, \quad u_0 = \frac{B}{2\mu} y (d - y), \quad B = \text{const} > 0. \tag{10}$$

We impose a perturbation on the flow by using connection of the source q_1. Finally, after taking account of the equations for the zeroth approximation we obtain an equation for the temperature perturbation

$$c_P \rho_0 u_0 \frac{\partial T_1}{\partial x} = \lambda \frac{\partial^2 T_1}{\partial y^2} + q_1. \tag{11}$$

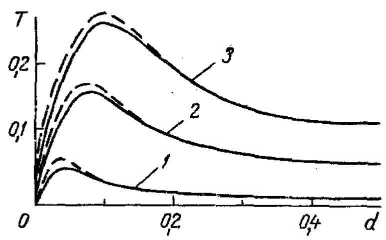

Fig. 1. Temperature distribution across the stream as a function of the distance to the entrance to the discharge chamber: 1) at a distance of 0.1th the discharge chamber length; 2) 0.5th the length; 3) at the end of the chamber; solid curves yield the temperature distribution in the case of homogeneous heat liberation, and the dashes when heat liberation is given by (12) (a constant temperature is given at the wall).

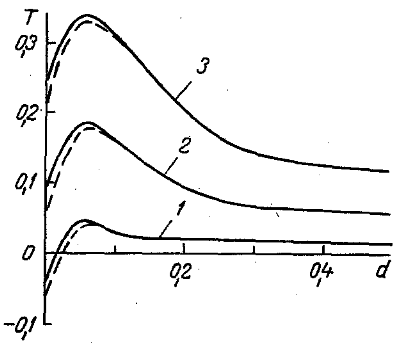

Fig. 2

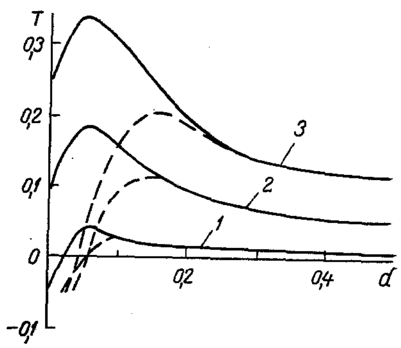

Fig. 3

Fig. 2. Temperature distribution across the stream at a distance of 0.1th the discharge chamber length from the entrance (1), 0.5th the length (2), at the end of the chamber (3); the heat liberation is given by (10) for the solid lines, and by (13) for the dashes (constant heat elimination is given at the wall).

Fig. 3. Temperature distribution across the stream at a 0.1th the length of the discharge chamber from the entrance (1), at 0.5th the length (2), at the chamber end (3); heat elimination for the solid lines is two times less than for the dashes, heat liberation is given by (12) (constant heat elimination is given at the wall).

NUMERICAL COMPUTATION OF THE TEMPERATURE FIELD

Equation (11) is reduced to dimensionless form by standard means and is solved numerically. The change in temperature T_1 is given in all the figures in fractions of the initial temperature T_0. It is initially assumed that a constant temperature is given at the walls while heat liberation is identical in the whole volume. Then a more correct problem was posed when the heat liberation per unit volume has the form

$$Q = Q_0\left(3 - \frac{2y}{0{,}03}\right), \quad y \leqslant 0{,}03; \quad Q = Q_0, \quad 0{,}03 < y < 0{,}97;$$

$$Q = Q_0\left[1 + 2\left(\frac{y - 0{,}97}{0{,}03}\right)\right], \quad y \geqslant 0{,}97. \qquad (12)$$

It is seen from Fig. 1 that even at the beginning of the discharge chamber an elevated temperature zone exists near the wall. Measurement is performed from the wall in fractions of the transverse chamber size. The temperature maximum grows as the gas mixture flows through the chamber, where utilization of the more correct heat liberation distribution will result in a still greater increase in the maximum and its approximation to the wall. The temperature distribution in the case of giving constant heat elimination from the wall is shown in Fig. 2. The case when the source distribution is

$$Q = Q_0\left(2 - \frac{y}{0{,}05}\right),\quad y \leqslant 0{,}05;\quad Q = Q_0,\quad 0{,}05 < y < 0{,}95;$$
$$Q = Q_0\left(1 - \frac{y - 0{,}95}{0{,}05}\right),\quad y \geqslant 0{,}95 \tag{13}$$

is shown by dashed lines. It is seen from Fig. 2 that an increase in the heat liberation near the wall results in raising the maximum temperature. In real systems the heat liberation zone is still narrower and more steep [1], which should apparently result in a still greater temperature difference between the center of the stream and the boundary layer. Unfortunately, a further increase in heat liberation near the wall will already result in spoiling the condition under which the equations have been deduced although the qualitative behavior of the temperature distribution curves is sufficiently conceivable from the above. The temperature distributions as a function of the heat elimination are represented in Fig. 3.

Summarizing, it can be said that the presence of a boundary layer results in a much stronger temperature rise occurring near the wall than at the center in the gas of heat liberation in a gas stream. This agrees qualitatively with experimental data for GDC [7]. Such a temperature field inhomogeneity can contribute to the development of a temperature-overheating instability.

NOTATION

u is the stream velocity; ρ is the stream density; x is the coordinate along the flow; P is the static pressure in the stream; μ is the viscosity coefficient; c_p is the specific heat for constant pressure; T is the temperature; λ is the heat conduction coefficient; q is the heat source function; R^* is the reduced gas constant; M is the Mach number; a_0 is the speed of sound at the entrance to the discharge chamber; d is the distance between electrodes, and y is the coordinate across the flow.

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